

# Reflection off Surfaces of Revolution: a Teachers' primer to the Introduction of the Cross-section Concept

**Marco Giliberti and Luca Perotti<sup>a</sup>**

Sezione Didattica della Fisica, Dipartimento di Fisica dell'Università di Milano, via  
Celoria 16, 20133 Milano Italy

<sup>a</sup>Department of Physics, Texas Southern University, Houston, Texas 77004 USA

**Abstract.** The differential cross-section for the reflection of light beams off rigid bodies obtained by the rotation of a generic derivable convex function is calculated. The calculation is developed using elementary notions of calculus and is therefore suitable for calculus oriented introductory undergraduate university physics courses. Three particular cases are presented as examples of the general procedure and of the physical properties and considerations about cross-sections they allow to discuss in class.

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## 1. Introduction

Cross-section is a fundamental conceptual tool in modern physics. As the “effective surface” the target presents to the probing radiation, be it material or electromagnetic, the cross-section contains all the information about the nature of the interactions between probes and target we can extract from the experiment, and which can be used to derive information on the target structure in the inverse scattering problem.

Unfortunately the first taste of this concept usually comes to the students in the case of the Geiger-Marsden experiment on the Rutherford scattering. The calculation of cross-sections for this and other physically relevant cases, such as deep inelastic scattering in Nuclear Physics, is usually rather long and often too difficult for modern undergraduate students; moreover the Rutherford scattering is a very special case, in that, as the Coulomb potential is a long-range one, its total cross section is infinite. Examples useful to clarify the geometric meaning of cross-section, if given, are therefore generally very few: typically scattering off rigid spheres, which has the drawback of being a very special case of isotropic scattering.

To overcome this difficulty, we have experimented with an approach where the concept of cross section is introduced through a class of interactions for which it is especially easy to grasp: that of reflection (according to the rules of geometrical optics) of light beams off surfaces having axial symmetry. Although the cross-sections thus calculated are of limited physical significance (but so are most classical mechanics examples), the interaction is of a kind closer to the student's everyday experience than those for physically relevant cases and can also easily be practically demonstrated in class.

Based on simple geometrical considerations, this approach allows an explicit calculation of several cross-sections with very little effort: namely that of finding the inverse function of a derivative (a similar approach, but within the frame of particle mechanics, and using a different procedure, can be found in Ref. [1]). The application of the formula allows the teacher ample material for classroom discussions about the concept of cross-section, thus providing a clear introduction to the concept, useful for further developments. The mathematics required is standard subject in most science university courses, but, as we observed in our in-class experimentation, the introductory physics students are still “green” enough in physics that intuitive geometrical examples as the ones here proposed can be both interesting and useful in fixing in their minds both the basic concepts and the difficulties, such as for example the fact that the inverse scattering problem is an arduous one, and often also an ill-posed one ‡.

While our original experimentation was conducted within the frame of particle scattering off rigid bodies, the formulation we present here is in terms of “optical” scattering, more suitable to contemporary views in fundamental physics education [2, 3]. As the arguments used are strictly geometrical, the difference between the two

‡ A simple example for the students is given by the Rutherford scattering itself: repulsive and attractive Coulomb potentials give the same cross section.

formulations is just in the language used and any teacher wishing to present the subject matter in a traditional course, will have no problem in reverting to a particle scattering description.

The paper is organized as follows: section 2 introduces through an example the concept of total cross-section; section 3 introduces the class of interactions we use as a model and through it illustrates the concept of differential cross-section; section 4 outlines the proposed approach to the calculation of the differential cross-section for the chosen class of interactions; section 5 presents three particular significant cases, namely that of ellipsoids, which reduces in the case of equal semi-axes to the classical example of spherical targets; the case of paraboloids, which give the same angular dependence of the differential cross-section as the Rutherford experiment and, finally, the case of targets generated by the rotation of an inverse sine curve which presents a curious similarity to our second example. In section 6 we discuss our approach and summarize the advantages of the proposed approach.

## 2. The concept of total cross-section

Suppose we have some *perfectly reflecting* solids, held fixed at some points of space, and imagine to shoot at them a well collimated light beam whose wavelength is short with respect to the dimensions of these solids, and which is diffused by the individual targets: see figure 1.

A first question we can ask ourselves is: what fraction of the beam hits the targets? Or, more precisely worded, what is the ratio of scattered energy flux to incoming energy flux? It is evident that the answer to the above question depends:

- a) on the number of targets per unity volume, that is on the density  $n$  of the targets,
- b) on the length  $h$  of the layer of targets,
- c) and on the section  $\sigma_T$  that each one of them shows to the beam.

This last quantity is what is called the total cross-section for the beam-target interaction.

Let us suppose, for simplicity, that:

- i) the beam has section  $S$ ;
- ii) the individual targets are sufficiently spaced from each other and  $h$  is small, so that  $nh\sigma_T \ll 1$  and the probability that one target covers another is negligible (thin target hypothesis).

In this case the total area shown to the beam by the target contained in the volume of base  $S$  and height  $h$  is  $nSh\sigma_T$ ; and the fraction  $P$  of the beam hitting the targets is given by its ratio to the beam section  $S$ :

$$P = \frac{nSh\sigma_T}{S} = nh\sigma_T \quad (1)$$

Provided the target is thin, in the sense of hypothesis ii), relation (1) may be regarded as a general definition of the total cross-section  $\sigma_T$ .

In the following we shall always assume hypothesis ii) to be verified. For simplicity we shall moreover only consider individual targets having axial symmetry and assume the beam to be shot at them along the direction of their axis.

### 3. The concept of differential cross section

As the individual targets are perfectly reflecting, the incoming beam is *deflected* by the interaction with the target. A second question we can therefore ask ourselves is: what fraction of the incoming beam is scattered by more than a given angle?

To answer this question, we can start considering a beam ray hitting one of the individual targets, at a distance  $b$  from the axis of the body ( $b$  is called impact parameter). The ray will be deflected according to the reflection laws: the incident ray, the perpendicular to the reflecting surface at the reflection point and the reflected ray all lie in the same plane and the incidence angle is equal to the reflection angle. Let us call  $\phi$  the scattering angle, that is the angle of deviation from the incident beam direction caused by the reflection; see figure 2.

One can see that if the target is convex (the second derivative of the generating curve is strictly positive), then the smaller the impact parameter  $b$  the larger is the scattering angle  $\phi$ . That means that all the rays within a disc of area  $\pi b^2$ , with center on the target axis, and perpendicular to their direction, will suffer a scattering angle greater than  $\phi$ ; see again figure 2. We can now answer our question by saying that the fraction of the incoming beam which is scattered through an angle greater than  $\phi$  is:

$$P_{>\phi} = nh\pi b^2 \quad (2)$$

In other words and keeping in mind equation (1), it can be said that the cross-section for scattering through an angle greater than  $\phi$  is:

$$\sigma_{>\phi} = \pi b^2 \quad (3)$$

We observe that equation (3) is not “fundamental” in that it relates the cross-section to the impact parameter  $b$  which, in a scattering experiment, where the positions of the individual targets are not known, is not a measurable quantity. Nonetheless this equation will be of great importance for the next considerations.

We now further refine the question to: what fraction of the beam is scattered through an angle between  $\phi$  and  $\phi + d\phi$ ?

The rays scattered through an angle between  $\phi$  and  $\phi + d\phi$  are given by those deflected through an angle greater than  $\phi$  minus those deflected more than  $\phi + d\phi$ . These are the rays that hit the fixed body on a cross surface of area  $|d\sigma| = \sigma_{>\phi} - \sigma_{>(\phi+d\phi)}$ ; the required fraction is then:

$$P(\phi) = nh|d\sigma| \quad (4)$$

After the scattering, these rays are contained into a solid angle of amplitude  $d\Omega$  given by the ratio between the area of the spherical zone  $Z$ , of figure 3, and  $r^2$ , that is:

$$d\Omega = \frac{2\pi(r \sin \phi)rd\phi}{r^2} = 2\pi \sin \phi d\phi \quad (5)$$

and therefore the the beam fraction scattered around the angle  $\phi$ , per unit solid angle, is:

$$P(\phi) = nh \frac{|d\sigma|}{d\Omega} \quad (6)$$

Equation (6) is of general interest and it is valid for all scattering experiments in the thin target hypothesis. Introducing the incoming flux per unit surface  $dP/dS$  and the scattered flux per unit sold angle  $dP/d\Omega$ , it can be recast in the more symmetric form:

$$\frac{dP}{d\Omega} = nhS \frac{|d\sigma|}{d\Omega} \frac{dP}{dS}. \quad (7)$$

The fundamental quantity  $|d\sigma|/d\Omega$  is called the differential cross-section. To better understand its physical meaning we can extend our description of the experiment to include the detection process. Consider a beam shot against a fixed target and an ideal detector of effective section  $A$ , located at an angle  $\phi$  at a distance  $R$  from the target and perpendicular to the scattered beam direction. In this way it detects all the radiation in the solid angle of amplitude  $\Omega \sim A/R^2$ . If  $\Omega$  is sufficiently small,  $P(\phi)$  can be considered to be constant over the surface  $A$ , and the ratio between the detected and incident beam flux is given by  $P(\phi)$  multiplied by  $\Omega$  which is:

$$P_{\Omega}(\phi) = nh \frac{|d\sigma|}{d\Omega} \frac{A}{R^2} \quad (8)$$

Equation (8) shows a clear way of calculating the differential cross-section from given experimental measures and a comparison between (6) and (8) helps students enlighten the conceptual meaning of this useful quantity.

#### 4. Detailed calculation

Let's consider the rigid solid produced by the complete rotation around the  $y$  axis of the increasing convex function  $y = f(x)$  with  $x$  between 0 and  $a$ ; figure 2. Our aim is to calculate the total and differential cross-sections for the reflection of a beam, incident on this fixed solid, in the direction of the  $y$  axis. The calculation of the total cross-section

$$\sigma_T = \pi a^2 \quad (9)$$

is straightforward. For what concerns the differential cross-section we make reference to figure 2: since the incident ray, the perpendicular to the surface and the reflection ray lie all on the same plane, the problem can be considered in the  $x - y$  plane. Let  $b$  be the impact parameter,  $t$  the tangent to the curve in the collision point,  $p$  the perpendicular,  $j$  the line of incidence and  $d$  the line of reflection. Now  $j$  is orthogonal to the  $x$  axis and  $p$  is orthogonal to  $t$ , therefore the angle  $\alpha$ , between  $t$  and the  $x$  axis, is equal to the angle  $\hat{i}$  between  $j$  and  $p$ , which is the incidence angle.

Keeping an eye on figure 2, it follows that the connection between the impact parameter  $b$  and the deflection angle  $\phi$  is:

$$g(b) \equiv \left. \frac{df}{dx} \right|_{x=b} = \tan \alpha = \tan \hat{i} = \tan \left( \frac{\pi - \phi}{2} \right) = \cot \frac{\phi}{2} \quad (10)$$

where  $\left. \frac{df}{dx} \right|_{x=b}$  is the derivative of  $f$  at point  $b$ .

We are now able to obtain the differential cross-section. The principal steps are the following:

**I.** Solve equation (10) with respect to  $b$ , that is: express  $b$  as a function of  $\phi$ , with the obvious geometric limitation  $0 \leq b \leq a$  due to the finite size of the target. In formulae, defining the inverse function  $g^{-1}$  through the equation  $g^{-1}(g(x)) = x$ :

$$\begin{cases} b(\phi) = g^{-1}\left(\cot \frac{\phi}{2}\right) \\ \pi - 2 \arctan(g(a)) \leq \phi \leq \pi - 2 \arctan(g(0)) \end{cases} \quad (11)$$

**II.** Calculate  $\sigma(\phi) = \pi b^2$ .

**III.** Differentiate  $\sigma(\phi)$  to obtain  $|d\sigma(\phi)|$ .

**IV.** Divide the found expression by  $d\Omega$  given by eq.(5).

## 5. Three examples

We present here, as examples of the procedure above, three cases we think significant, together with sample discussions of some key points of the theory; teachers can choose among them, make up their own comments, or invent other examples more suited to their individual aims.

**1) ellipsoids, paraboloids and spheres:** let's consider the ellipsoid obtained by the rotation, around the  $y$  axis, of the function (shown in Fig. 4 a)

$$f(x) = -c\sqrt{1 - \left(\frac{x}{a}\right)^2}; \quad 0 \leq x \leq a, \quad (12)$$

with  $a$  and  $c$  the two semi-axes of the generating ellipse (this example is discussed in detail in [4]).

From step I, equation (11), we get

$$b^2 = a^2 \frac{\cot^2\left(\frac{\phi}{2}\right)}{\frac{c^2}{a^2} + \cot^2\left(\frac{\phi}{2}\right)}, \quad (13)$$

and then (steps II., III., and IV.):

$$\begin{cases} \frac{|d\sigma(\phi)|}{d\Omega} = \frac{a^2}{4} \left( \frac{ac}{c^2 \sin^2\left(\frac{\phi}{2}\right) + a^2 \cos^2\left(\frac{\phi}{2}\right)} \right)^2 \\ 0 \leq \phi \leq \pi \end{cases} \quad (14)$$

From the obvious relations:  $\int d\Omega = 4\pi$  and  $\int \frac{d\sigma}{d\Omega} d\Omega = \sigma_T$ , where both integrals are on the whole solid angle (or on "all directions"), and from equation (14), one gets immediately the trivial result  $\sigma_T = \pi a^2$ . This can be a useful check for most students.

A simple change of signs in eq. (12) gives us the equation of a hyperboloid (shown in Fig. 4 b):

$$f(x) = c\sqrt{1 + \left(\frac{x}{a}\right)^2}; \quad 0 \leq x \leq a, \quad (15)$$

which results in the cross-section

$$\left\{ \begin{array}{l} \frac{|d\sigma(\phi)|}{d\Omega} = \frac{\tilde{a}^2}{4} \left( \frac{\tilde{a}c}{c^2 \sin^2(\frac{\phi}{2}) - \tilde{a}^2 \cos^2(\frac{\phi}{2})} \right)^2 \\ \pi - 2 \arctan\left(\frac{ca}{\tilde{a}} \frac{1}{\sqrt{\tilde{a}^2 + a^2}}\right) \leq \phi \leq \pi \end{array} \right. \quad (16)$$

where, as the curve (15) is not bound in  $x$ , we had to distinguish between the target size parameter  $a$  (introduced as to avoid an infinite total cross-section) and the curve parameter  $\tilde{a}$ .

Changing the ratio  $c/a$  for the ellipsoid, or  $c/\tilde{a}$  for the hyperboloid, changes the angular distribution of the scattered rays. In particular, for  $c = a$ , eq. (14) reduces the well known expression of the differential cross-section of a rigid sphere [5]  $|d\sigma(\phi)|/d\Omega = a^2/4$ , which depends only on the sphere radius  $a$ , which is a constant, but is independent of  $\phi$ . This means that after the collision with a sphere the rays are isotropically scattered, that is: they are deflected to every angle with equal probability. This result depends on the particular interaction here considered.

In a scattering experiment we do not usually know the *exact* nature and composition of the target, and the aim of the experiment is to deduct the missing information, be it the potential for a target composed of force centers, or the generating curve  $f(x)$  in the cases here discussed. Direct reconstruction of the unknown scattering potential, or of the generating curve, is an arduous task, on which whole books have been written [6]; on the other hand, partial information on the nature of the target can help reduce the range of possible interactions. This either makes a “trial and error” approach, such as was used in the interpretation of early scattering experiments, more feasible, or even allows univocal direct solution of the problem [7].

The simplest example one can give -that of isotropic scattering- already allows for the highlight of some necessary subtleties: if in a scattering experiment the detector gives the same reading at all angles, independently from the physical properties of the incident beam, such as, for example, the energy (such a dependence would suggest a target composed of force centers [5]), or the electric charge carried by the beam, from which it could a priori depend, we have strong indications that the interaction between the beam and the target is a reflection off the surface of the individual target, and that the target is “made of” rigid spheres, impenetrable by the beam.

It seems a simple and straightforward procedure, but we must note that both the angular dependence, *and also its range of validity* are important to determine the scattering surface: a differential cross-section which over a given range of angles has the constant value  $a^2/4$  can be caused by targets generated by any curve having a derivative of the form [8]

$$g(x) = \sqrt{\frac{\gamma^2 + x^2}{\beta^2 - x^2}}, \quad 0 \leq x \leq \beta, \quad \gamma^2 + \beta^2 = a^2, \quad (17)$$

but it is zero outside the range  $0 \leq \phi \leq \pi - 2 \arctan(|\gamma/\beta|)$ , while the differential cross-section for spheres is constant over the whole range  $0 \leq \phi \leq \pi$ .

More examples of reconstruction of hard reflecting surfaces of revolution from observed cross sections can be found in Ref. [1].

**2) Paraboloids:** let us consider the solid obtained by the rotation of the curve [9] (shown in Fig. 4 c)

$$f(x) = \frac{x^2}{c}; \quad 0 \leq x \leq a. \quad (18)$$

Reminding equation (9), the total cross section is again  $\sigma_T = \pi a^2$ . For the differential cross-section, step I. (equation (11)) instead gives us:

$$b = \frac{c}{2} \cot\left(\frac{\phi}{2}\right) \quad (19)$$

The following steps (steps II.to IV.) then give us:

$$\left\{ \begin{array}{l} \frac{|d\sigma|d\Omega}{=4} = \frac{\pi c^2}{4} \frac{\cos(\frac{\phi}{2})d\phi}{\sin^3(\frac{\phi}{2})} \frac{1}{2\pi \sin \phi d\phi} = \frac{c^2}{16} \text{cosec}^4\left(\frac{\phi}{2}\right). \\ \pi - 2 \arctan\left(2\frac{a}{c}\right) \leq \phi \leq \pi \end{array} \right. \quad (20)$$

Note that, contrary to the previous case, where the ratio  $c/a$  influences the angular distribution of the scattered beam, here  $c$  is just a scale factor: the angular distribution is independent from it.

This result shows some similarities with the Rutherford cross-section [10, 11]:

$$\frac{|d\sigma_R|}{d\Omega} = \frac{K^2}{16} \text{cosec}^4\left(\frac{\phi}{2}\right); \quad K = \frac{Ze\rho_q}{4\pi\epsilon_0\rho_{K_0}}, \quad (21)$$

where  $Ze$  is the target charge,  $\rho_q$  and  $\rho_{K_0}$  are the charge and kinetic energy densities of the incident beam, and  $\epsilon_0$  is the vacuum permittivity [12]. Clearly, the same angular dependence appears; moreover, both of the cross sections are not valid for small angles, but for very different reasons: in the present case because of the discontinuity of  $f(x)$  in  $x = a$ , and in the Rutherford case because when the impact parameter is large the shielding effect of the atoms' electronic cloud is no more negligible. How can we then distinguish the two cases? First of all, eq. (20) is only valid when the incidence direction is that of the symmetry axis of the individual targets; a rotation of the target with respect to the incident beam would result in a different result. More important, the differential cross-section given by eq. (20) shows no dependence from the kinetic energy carried by the incident beam (the variable in the Rutherford experiment which can be most easily changed): we would therefore have to imagine different scattering surfaces for beams having different kinetic energies: a highly unlikely circumstance.

**3) the arcsin as the symmetric scatterer to the Paraboloid:** let's finally consider the surface obtained by the rotation, around the  $y$  axis, of the function (shown in Fig. 4 d)

$$f(x) = \tilde{c} \arcsin \frac{x}{a}; \quad 0 \leq x \leq a. \quad (22)$$

From step I, equation (11), we get

$$b^2 = a^2 - \tilde{c}^2 \tan^2\left(\frac{\phi}{2}\right) \quad (23)$$



then (steps II.- IV.):

$$\begin{cases} \frac{|d\sigma|}{d\Omega} = \frac{\tilde{c}^2}{4} \frac{1}{\cos^4\left(\frac{\phi}{2}\right)} = \frac{\tilde{c}^2}{4} \sec^4\left(\frac{\phi}{2}\right) \\ 0 \leq \phi \leq \pi - 2 \arctan\left(\frac{\tilde{c}}{a}\right), \end{cases} \quad (24)$$

which is a mirror image of our previous example eq. (20), obtained by the substitutions  $\phi \rightarrow \pi - \phi$  and  $c \rightarrow 2\tilde{c}$ . The transformation between the two ranges in  $\phi$  follows the same simple rule, as for  $\alpha \in (0, \pi/2)$ ,

$$\frac{\pi}{2} - \arctan \alpha = \arctan\left(\frac{1}{\alpha}\right). \quad (25)$$

## 6. Discussion and Conclusions

The concept of cross-section in the case of reflection off the surface of a solid has been introduced on the basis only of simple statistical and geometric considerations. This choice has been made because of didactic reasons: with this approach, the intuition is helped by the “material” existence of the cross-sections both total and differential (which are real parts of the rigid surface of the body).

In physically more significant cases, as e. g. the Rutherford scattering itself, the interaction is between the incoming beam and the force field of the target. It is true that in these cases the total cross-section can be geometrically conceived as the “effective surface” presented by the target force field to the incoming beam and the differential cross section as the “part” of that “surface” scattering the beam in a given direction, but this only in an abstract sense, which usually is not easy for the students to grasp: for example, even in the simple geometrical case of a light beam randomly scattered by a rough surface, the differential cross section, even if mathematically defined, cannot be associated with any geometrically definite part of the scattering surface.

Besides this difficulty, the general approach, in which the scattering is introduced from the beginning in its abstract meaning, often allows students only to discuss existing experimental data, thus missing important relations and theoretical considerations, an understanding of which can be gained by the direct calculation of several simple cross-sections.

In conclusion, this paper gives a simple formula to calculate with little effort many different cross-sections, three of which are explicitly given (in the case of a rigid ellipsoid, in that of a rigid paraboloid, and in that of the solid generated by the rotation of an inverse sine curve). The formula itself is of limited utility because of the very simple interaction taken into account but, at the introductory level, it can help students understand the link between experiments and theoretical explanations.

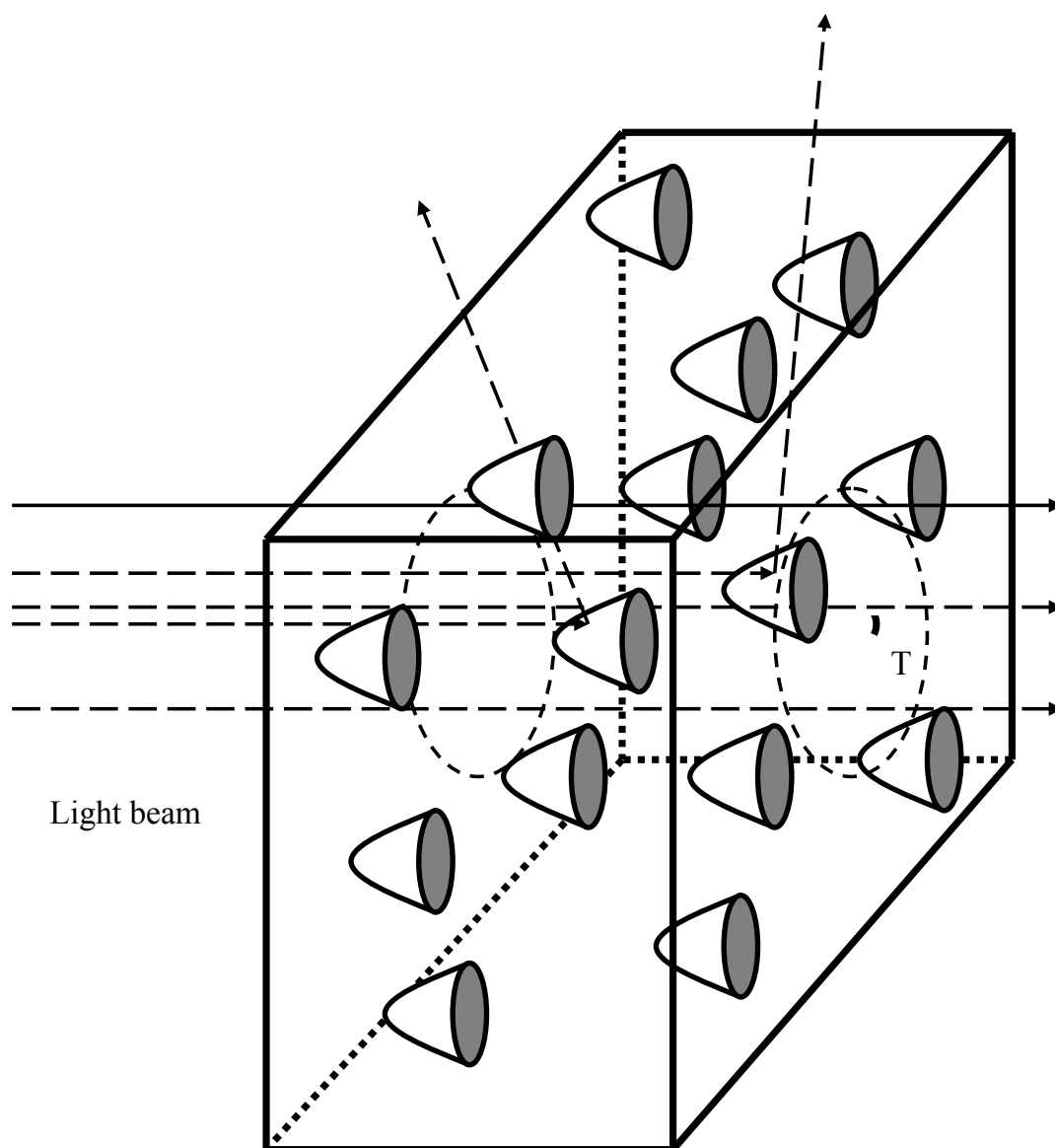
An extended case study, about the students' understanding of the cross-section concept, following the ideas outlined in the present paper, is under way and will be presented in a forthcoming paper.

## Acknowledgments

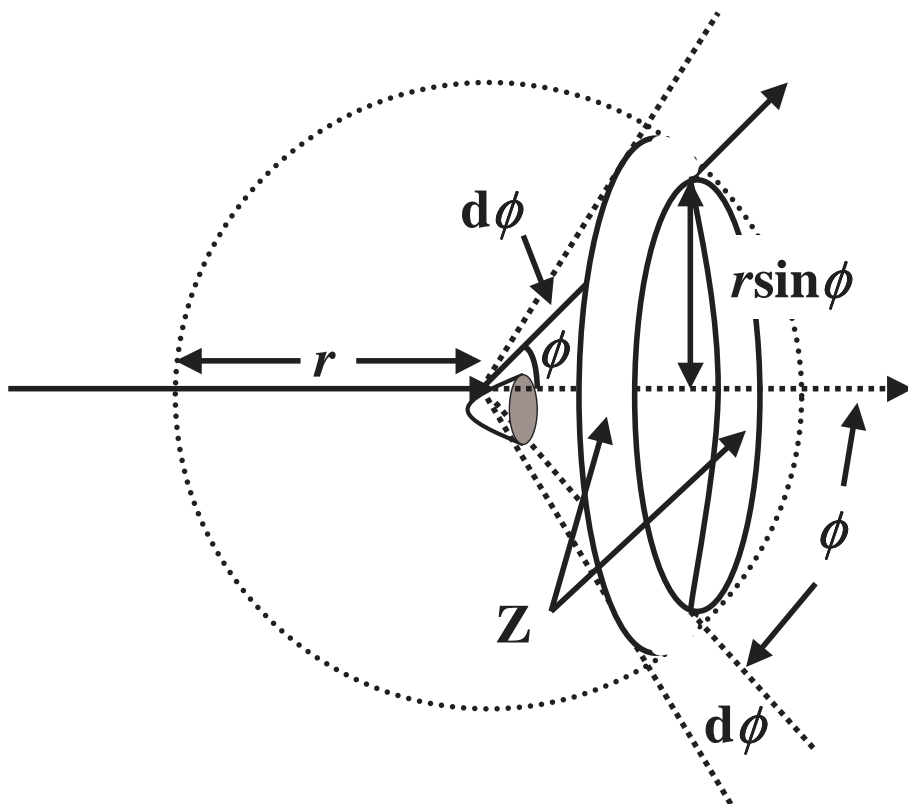
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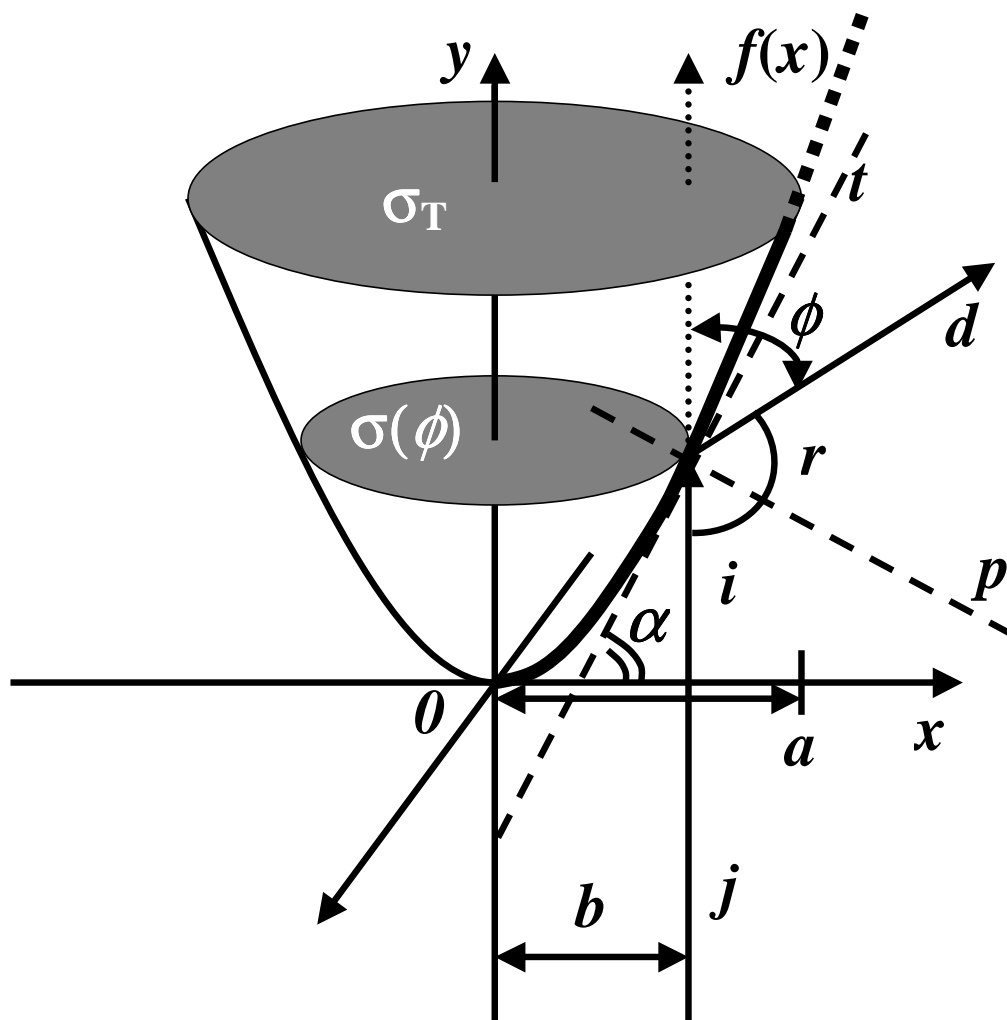
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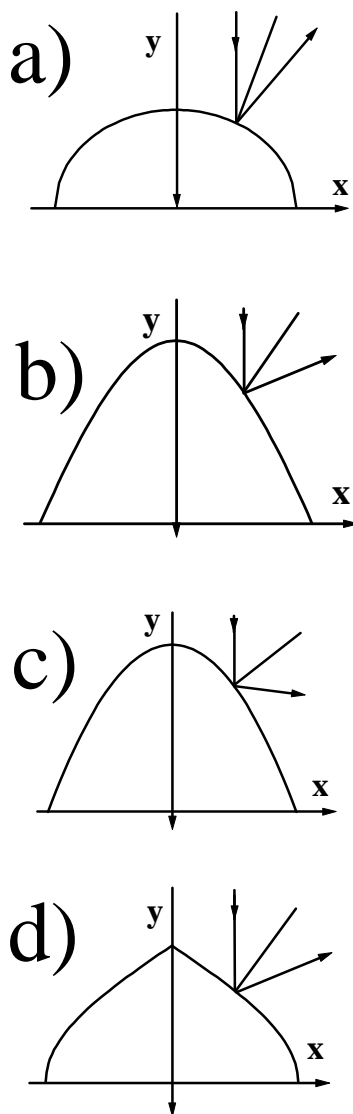
**Figure 1.** A collimated beam is shot against a thin target of reflecting solids, each one of total cross-section  $\sigma_T$ , fixed at some points of the space. Most of the beam will be undeflected but part of it will be scattered by the solids.



**Figure 2.** A sharp-edged solid is given by the complete rotation around the  $y$  axis of the increasing convex function  $y = f(x)$  with  $x$  between 0 and  $a$ . A beam ray, directed along the  $y$  axis, is reflected and scattered by the solid through the angle  $\phi$ :  $b$  is the impact parameter,  $t$  the tangent and  $p$  the perpendicular to the solid at the collision point,  $c$  and  $d$  are respectively the incident and the reflected ray.



**Figure 3.** The beam rays scattered between  $\phi$  and  $\phi + d\phi$  are contained into a solid angle of amplitude given by the ratio between the area  $2\pi r^2 \sin \phi d\phi$ , of the spherical zone  $Z$ , and  $r^2$ .



**Figure 4.** The curves generating the reflecting solids of our examples: a) an ellipse; b) an hyperbola; c) a parabola; d) an inverse sine.